Pergamon

PII: S0021-8928(97)00026-9

J. Appl. Maths Mechs, Vol. 61, No. 2, pp. 195–202, 1997

0021-8928/97 \$24.00+0.00

SCATTERING OF HYDROACOUSTIC WAVES BY A NARROW CRACK IN AN ELASTIC PLATE†

I. V. ANDRONOV and B. P. BELINSKII

St Petersburg

(Received 2 November 1995)

The scattering of waves by an infinite elastic plate, which covers an acoustic half-space and is weakened by a crack in the form of an infinitely long cut of finite width with parallel edges is considered. The scattered field is expressed in terms of the solution of an integro-algebraic system of equations on the crack. The logarithmic characteristic of the kernel enables Bubnov's method with a basis containing Chebyshev polynomials of the first kind to be used for the numerical analysis. Particular attention is given to the asymptotic investigation of the scattering diagram and the amplitudes of the surface waves for a narrow crack and a thin plate. A comparison with the well-known model of a point crack enables the range of parameters of the problem where the point model is applicable to be indicated. © 1997 Elsevier Science Ltd. All rights reserved.

The scattering of acoustic waves by an elastic plate, weakened by one or more point cracks, was considered previously in [1, 2]. Scattering by a plate reinforced with point ribs has been considered by many researchers. An explicit form of the solution has been obtained [3] and a numerical analysis of the effective scattering cross-section has been carried out. A review of foreign research devoted to low-frequency scattering by plates with inhomogeneities can be found in [4]. More-complex scattering problems can be reduced to integral equations [5] which can then be investigated asymptotically.

In all the models of plates with point inhomogeneities the problem arises of the conditions of applicability. An analysis of the possibility of replacing actual inhomogeneities by point models is essential. The problem of scattering by a low protruding rib was investigated in [6] and the conditions of applicability were obtained for the point-rib model.

Below we establish that certain features of the field scattered by a narrow crack are not reproduced in the point model.

1. FORMULATION OF THE PROBLEM

We will consider a system consisting of a homogeneous acoustic half-space $\{y > 0\}$, bounded by a thin elastic plate $\{|x| > a, y = 0\}$ with a crack $\{|x| < a\}$. The field in the system is described by the Helmholtz equation

$$\Delta U + k^2 U = 0, \quad y > 0 \tag{1.1}$$

The time dependence is taken in the form $\exp(-i\omega t)$. The oscillations of the plate are described by Kirchhoff's model [7]

$$\left(D\frac{d^4}{dx^4} - \rho\omega^2 h\right)\xi + U = 0, \quad y = 0, \ |x| > a; \ \xi = \frac{1}{\rho_0\omega^2}\left.\frac{\partial U}{\partial y}\right|_{y=0}, \quad D = \frac{Eh^3}{12(1-\sigma^2)} \tag{1.2}$$

Here ξ is the buckle of the plate, D is the cylindrical stiffness of the plate, E is Young's modulus, σ is Poisson's ratio, h is the plate thickness, and ρ and ρ_0 are the density of the plate and the acoustic medium.

The contact conditions on the crack edges express the fact that there are no shear forces and bending moments, and have the form

$$\frac{d^n\xi}{dx^n}(\pm a) = 0, \quad n = 2, 3 \tag{1.3}$$

†Prikl. Mat. Mekh. Vol. 61, No. 2, pp. 202–209, 1997.

Dirichlet's condition

$$U = 0, \quad y = 0, \, |x| < a \tag{1.4}$$

is satisfied on the crack.

Meixner's conditions are imposed at the points $\{x = \pm a, y = 0\}$, which guarantees that the energy in the acoustic medium is finite.

The wave field is excited by a certain spatial wave incident from the half-space, or a surface wave travelling along the plate from infinity. The field U^g, which would occur if there was no crack, can easily be obtained. When a plane wave

$$U^i = \exp\big(ik(x\cos\varphi_0 - y\sin\varphi_0)\big) \tag{1.5}$$

is incident, it consists of incident and reflected waves

$$U^g = U^i + R(\varphi_0)\exp\big(ik(x\cos\varphi_0 + y\sin\varphi_0)\big) \tag{1.6}$$

$$R(\varphi_0) = -\frac{\overline{L(\varphi_0)}}{L(\varphi_0)}, \quad L(\varphi_0) = ik\sin\varphi_0(k^4\cos^4\varphi_0 - k_0^4) + \nu, \quad k_0^4 = \frac{\rho h\omega^2}{D}, \quad \nu = \frac{\rho_0\omega^2}{D}$$

Here the bar denotes the complex conjugate, and we have introduced standard notation [1].

The correction to U^g will be called the scattered field. The scattered field $U^s = U - U^g$ will satisfy the radiation condition, i.e. it should not contain arriving waves. As will be shown below, at considerable distances from the crack the scattered field consists of a diverging cylindrical wave and two surface waves, concentrated close to the plate. The purpose of this paper is to construct the field U^s. We will assume that the crack width is small compared with the wave length of the incident field.

2. REDUCTION OF THE PROBLEM TO INTEGRAL EQUATIONS

We will introduce Green's function G of the boundary-value problem for a homogeneous plate

$$(\Delta + k^2)G = -\delta(x - x_0, \ y - y_0)$$

$$\left(\frac{d^4}{dx^4} - k_0^4\right)\frac{\partial G}{\partial y} + \nu G = 0$$

Then, using the second Green's formula we can obtain a representation for the scattered field

$$\begin{aligned}-U^s(x_0, y_0) = \int_{-a}^{a} G(x, 0, x_0, y_0)\frac{\partial U(x,0)}{\partial y}dx + D\xi(a)G_{yxxx}(a, 0, x_0, y_0) - \\ -D\xi(-a)G_{yxxx}(-a, 0, x_0, y_0) - D\xi_x(a)G_{yxx}(a, 0, x_0, y_0) + D\xi_x(-a)G_{yxx}(-a, 0, x_0, y_0)\end{aligned} \tag{2.1}$$

(derivatives are denoted by the appropriate subscripts).

Hence, if we know the normal derivative of the total field $\phi(x) \equiv \partial U(x, 0)/\partial y$ on the crack, and we know the overall displacements $\xi(\pm a)$ and the angles of rotation $\xi_x(\pm a)$ of the plate edges, the scattered field U^s can be calculated from (2.1). This field automatically satisfies Eqs (1.1) and (1.2) and the radiation condition for arbitrary $\phi(x)$, $\xi(\pm a)$ and $\xi_x(\pm a)$. Boundary condition (1.4) and contact conditions (1.3) lead to a system of equations in the function $\Phi(x)$ and the constants $\xi(\pm a)$ and $\xi_x(\pm a)$

$$\begin{aligned}\int_{-a}^{a} G(x, 0, x_0, y_0)\phi(x)dx + D\xi(a)G_{yxxx}(a, 0, x, x_0, 0) - D\xi(-a)G_{yxxx}(-a, 0, x, x_0, 0) - \\ -D\xi_x(a)G_{yxx}(a, 0, x_0, 0) + D\xi_x(-a)G_{yxx}(-a, 0, x_0, 0) = U^g(x_0, 0), \ -a < x_0 < a\end{aligned} \tag{2.2}$$

$$\frac{1}{D}\int_{-a}^{a}\frac{\partial^{n+1}G(x,0,x_0,0)}{\partial y_0\partial x_0^n}\phi(x)dx+\xi(a)\frac{\partial^{5+n}G(a,0,\pm(a+0),0)}{\partial y\partial y_0\partial x^3\partial x_0^n}-$$
$$-\xi(-a)\frac{\partial^{5+n}G(-a,0,\pm(a+0),0)}{\partial y\partial y_0\partial x^3\partial x_0^n}-\xi_x(a)\frac{\partial^{4+n}G(a,0,\pm(a+0),0)}{\partial y\partial y_0\partial x^2\partial x_0^n}+ \tag{2.3}$$
$$+\xi_x(-a)\frac{\partial^{4+n}G(-a,0\pm(a+0),0)}{\partial y\partial y_0\partial x^2\partial x_0^n}=\nu\frac{\partial^n}{\partial x^n}\xi^g(\pm a),\ \ n=2,3$$

The class of functions within which the solution $\Phi(x)$ must lie is determined by the Meixner conditions

$$\phi(x)=O\left((x\mp a)^{\delta-1}\right),\ \ \delta>0$$

The operator of the integro-algebraic system (2.2), (2.3) is expressed in terms of Green's function $G(x,y,x_0,y_0)$. This Green's function is well known and can be obtained by the Fourier method. On the boundary $\{y = 0\}$ we obtain for the traces G and $\partial G/\partial y_0$

$$G(x,0,x_0,y_0)=\frac{1}{2\pi}\int_{-\infty}^{+\infty}\exp\left(i\lambda(x-x_0)-\sqrt{\lambda^2-k^2}y_0\right)\frac{\left(\lambda^4-k_0^4\right)}{l(\lambda)}d\lambda \tag{2.4}$$

$$G_y(x,0,x_0,y_0)=-\frac{\nu}{2\pi}\int_{-\infty}^{+\infty}\exp\left(i\lambda(x-x_0)-\sqrt{\lambda^2-k^2}y_0\right)\frac{d\lambda}{l(\lambda)} \tag{2.5}$$

$$\left(l(\lambda)=(\lambda^4-k_0^4)\sqrt{\lambda^2-k^2}-\nu\right)$$

The zeros of the function $l(\lambda)$ at the points $\lambda = \pm\varkappa$ are circumvented by the integration contour below and above, respectively.

To calculate the higher-order derivatives, which occur in the integro-algebraic system, we need to regularize the integrals, which can be done by methods which are standard for the theory of boundary-contact problems [2]. The integrand in the representation of the kernel (2.4) for large λ will be $O(1/|\lambda|)$ and hence the kernel of integral equation (2.2) has a logarithmic singularity. It can be shown that the remaining functions involved in the system are bounded.

3. NUMERICAL PROCEDURE

By introducing the new unknowns

$$[\xi_x]=\rho_0\omega^2\left(\frac{\partial\xi(a)}{\partial x}-\frac{\partial\xi(-a)}{\partial x}\right),\ \ \{\xi_x\}=\rho_0\omega^2\left(\frac{\partial\xi(a)}{\partial x}+\frac{\partial\xi(-a)}{\partial x}\right)$$
$$[\xi]=\rho_0\omega^2(\xi(a)-\xi(-a)),\ \ \{\xi\}=\rho_0\omega^2(\xi(a)+\xi(-a))$$

system (2.2), (2.3) can be rewritten in symmetrical form. When carrying out the numerical procedure this enables the system to be split into two, corresponding to the parts of the field that are even and odd in x.

We will use Bubnov's method [8] for the numerical analysis. It is convenient to choose Chebyshev polynomials of the first kind $T_p(x/a)$, divided by the square root $\sqrt{(a^2-x^2)}$, as the basis. This basis is the basis of the eigenfunctions of the integral operator of the first kind with a logarithmic kernel [9].

We expand the function $\Phi(x)$ in series (everywhere henceforth, unless otherwise stated, summation is carried out from $p = 0$ to $p = \infty$)

$$\phi(x)=\Sigma\phi_p\frac{T_p(x/a)}{\sqrt{a^2-x^2}} \tag{3.1}$$

then substitute into the system and project the integral equation onto $T_q(x/a)$. As a result we obtain two infinite algebraic systems for determining the coefficients of expansion (3.1) and the constants $[\xi_x]$, $\{\xi_x\}$, $[\xi]$ and $\{\xi\}$. For brevity we will only give the system corresponding to the part of the field that is even in x

$$\Sigma A_{2p2q}\phi_{2p}+B_{2q}[\xi_x]+C_{2q}\{\xi\}=u_{2q},\ \ q=0,1,\ldots$$

$$\sum B_{2p}\phi_{2p}+\frac{D_0+E_0}{2}[\xi_x]-\frac{E_1}{2}\{\xi\}=-\frac{D}{2}\left(\xi^g_{xx}(a)+\xi^g_{xx}(-a)\right)$$

$$\sum C_{2p}\phi_{2p}-\frac{E_1}{2}[\xi_x]-\frac{D_2-E_2}{2}\{\xi\}=\frac{D}{2}\left(\xi^g_{xxx}(a)-\xi^g_{xxx}(-a)\right) \tag{3.2}$$

The elements of the matrices of these systems are expressed by double integrals over a square $-a < x < a$, $-a < t < a$, or after substitution of the representation for the kernel and evaluation the integrals with respect to x and t, by integrals along the semiaxis

$$A_{pq}=i^{p+q}\pi\int_0^{+\infty}J_p(a\lambda)J_q(a\lambda)\frac{\lambda^4-k_0^4}{l(\lambda)}d\lambda$$

$$B_{2p}=(-1)^{p+1}\int_0^{+\infty}J_{2p}(a\lambda)\cos(a\lambda)\frac{\lambda^2}{l(\lambda)}d\lambda,\quad B_{2p+1}=(-1)^{p+1}\int_0^{+\infty}J_{2p+1}(a\lambda)\sin(a\lambda)\frac{\lambda^2}{l(\lambda)}d\lambda \tag{3.3}$$

$$C_{2p}=(-1)^{p}\int_0^{+\infty}J_{2p}(a\lambda)\sin(a\lambda)\frac{\lambda^3}{l(\lambda)}d\lambda,\quad C_{2p+1}=(-1)^{p}\int_0^{+\infty}J_{2p+1}(a\lambda)\cos(a\lambda)\frac{\lambda^3}{l(\lambda)}d\lambda$$

$$D_j=\frac{1}{2\pi}\int_{-\infty}^{+\infty}\left\{\frac{k_0^4}{\nu}\sqrt{\lambda^2-k^2}+1\right\}\frac{(i\lambda)^j}{l(\lambda)}d\lambda,\quad j=0,2 \tag{3.4}$$

$$E_j=\frac{1}{2\pi}\int_{-\infty}^{+\infty}e^{2ia\lambda}\left\{\frac{k_0^4}{\nu}\sqrt{\lambda^2-k^2}+1\right\}\frac{(i\lambda)^j}{l(\lambda)}d\lambda,\quad j=0,1,2$$

To justify the truncation method we will estimate the behaviour of the elements of matrix (3.3) with respect to numbers. To do this we calculate the asymptotic form $1/|\lambda|$ of the symbol $(\lambda^4-k_0^4)/l(\lambda)$ at infinity

$$A_{pq}=i^{p+q}\frac{\pi}{2}\left\{\frac{\delta_q^p}{2p}+\int_0^{+\infty}J_p(a\lambda)J_q(a\lambda)\left(\frac{\lambda^4-k_0^4}{l(\lambda)}-\frac{1}{\lambda}\right)d\lambda\right\},\quad p+q>0 \tag{3.5}$$

($\delta_p^q = 1$ when $p = q$ and $\delta_p^q = 0$ when $p \neq q$). Estimates of the integrals (3.5) can be obtained in a similar way [10] with additional consideration of the contribution of the pole at the point $\lambda = \varkappa$. We will give the final expression

$$|A_{pq}|<C\left(\frac{a}{2}\right)^{2-9/(p+q+4)}(p!q!)^{-3/(p+q+4)}\left[1+\left(\frac{a}{2}\right)^{6/(p+q+4)}\right] \tag{3.6}$$

The limit (3.6) enables us to conclude that for each fixed p the sum of the off-diagonal elements $S_p = \Sigma_{q\neq p}|A_{pq}|$ is finite and decreases as p increases. Similar limits for the integrals B_p and C_p show that $\Sigma|B_p|$ and $\Sigma|C_p|$ are finite. These properties of the matrix prove that the truncation method can be used [11].

4. THE ASYMPTOTIC FORMS OF THE RADIATION PATTERN AND THE AMPLITUDES OF THE SURFACE WAVE

The scattered field U^s can be represented at considerable distances from the crack in the form of the sum of a cylindrical diverging wave U^0 and surface waves $U^{\pm}$. The expression for the cylindrical wave

$$U^0=\sqrt{\frac{2\pi}{kr}}e^{ikr-i\pi/4}\Psi(\varphi)$$

$$\Psi(\varphi)=\frac{1}{2\pi}\frac{k\sin\varphi}{ik\sin\varphi(k^4\cos^4\varphi-k_0^4)+\nu}\left\{\left(k^4\cos^4\varphi-k_0^4\right)\int_{-a}^{a}e^{-ikx\cos\varphi}\phi(x)dx+\right. \tag{4.1}$$

$$+ik^2\cos^2\varphi\sin(ka\cos\varphi)\{\xi_x\}-k^2\cos^2\varphi\cos(ka\cos\varphi)[\xi_x]-$$
$$-k^3\cos^3\varphi\sin(ka\cos\varphi)\{\xi\}-ik^3\cos^3\varphi\cos(ka\cos\varphi)[\xi]\big\}$$

is obtained after applying the saddle-point method to the integral in (2.1). We have introduced the polar coordinates $r = \sqrt{(x_0^2 + y_0^2)}$, $\varphi = \mathrm{arctg}(y_0/x_0)$ here. Expressions for the surface waves

$$U^{\pm} = A_{\pm}\exp\left(\pm i\varkappa x_0 - \sqrt{\varkappa^2 - k^2}\,y_0\right)$$
$$A_{\pm} = \frac{-i}{5\varkappa^4 - 4k^2\varkappa^2 - k_0^4}\left\{\frac{\nu}{\varkappa}\int_{-a}^{a} e^{\mp i\varkappa x}\phi(x)dx - \varkappa\sqrt{\varkappa^2 - k^2}\left(\cos(\varkappa a)[\xi_x] + \varkappa\sin(\varkappa a)\{\xi\}\right)\mp\right. \tag{4.2}$$
$$\left.\mp i\varkappa\sqrt{\varkappa^2 - k^2}\left(\varkappa\cos(\varkappa a)[\xi] - \sin(\varkappa a)\{\xi_x\}\right)\right\}$$

are obtained by taking the residue at the point $\lambda = \pm\varkappa$.

We will consider the case of a narrow crack, i.e. we will obtain the asymptotic form of the pattern Ψ and the amplitudes $A^{\pm}$ for $ka \ll 1$. Using the limit (3.6) it can be shown that the off-diagonal elements of the matrix A_{pq} are asymptotically small. The right-hand sides and the elements B_p and C_p tend to zero when $p > 0$. Hence, system (3.2) can be truncated with respect to the first equation for small ka. Hence we have (the tilde denotes the leading term of the asymptotic form when $ka \ll 1$ and γ is Euler's constant)

$$\tilde{A}_{00}\phi_0 - \tilde{B}_0[\xi_x] = \pi U^g(0),\quad -\tilde{B}_0\phi_0 + D_0[\xi_x] = -D\xi^g_{xx}(0),\quad D_2[\xi] = -D\xi^g_{xxx}(0) \tag{4.3}$$

$$\tilde{B}_0 = \int_0^{+\infty}\frac{\lambda^2 d\lambda}{l(\lambda)},\quad \tilde{A}_{00} = -\pi\left(A - \frac{\pi i}{2} - \frac{\nu}{2}I\right)$$
$$A = \ln\frac{ka}{4} + \gamma,\quad I = \int_{-\infty}^{+\infty}\frac{d\lambda}{l(\lambda)\sqrt{\lambda^2 - k^2}} \tag{4.4}$$

When oscillations of the system are excited by a plane wave (1.5) incident from the acoustic half-space, the right-hand sides of the system of equations (4.3) depend on the angle of incidence φ_0. It can be shown that the constants $\{\xi_x\}$ and $\{\xi\}$ are bounded, while the coefficients of expansion (3.1) are asymptotically small when $p > 0$. By calculating the coefficient ϕ_0 and the constants $[\xi_x]$ and $[\xi]$ from (4.3), using the accurate expressions (4.1) and (4.2), we obtain asymptotic expansions of the radiation pattern and the amplitudes

$$\Psi(\varphi,\varphi_0) = \frac{i}{\pi}\frac{k^2\sin\varphi\sin\varphi_0}{L(\varphi)L(\varphi_0)}\left\{\frac{k^4}{D_0}\cos^2\varphi\cos^2\varphi_0 - \frac{k^6}{D_2}\cos^3\varphi\cos^3\varphi_0 +\right.$$
$$+\frac{1}{\tilde{A}_{00}}\left(\pi^2 L_1(\varphi)L_1(\varphi_0) - \pi\frac{k^2\tilde{B}_0}{D_0}(\cos^2\varphi L_1(\varphi_0) + L_1(\varphi)\cos^2\varphi_0)+\right.$$
$$\left.\left.+\frac{k^4\tilde{B}_0^2}{D_0^2}\cos^2\varphi\cos^2\varphi_0\right) + \ldots\right\} \tag{4.5}$$
$$A_{\pm} = -\frac{2k\sin\varphi_0}{L(\varphi_0)}\frac{1}{5\varkappa^4 - 4k^2\varkappa^2 - k_0^4}\left\{\varkappa\sqrt{\varkappa^2 - k^2}\frac{k^2}{D_0}\cos^2\varphi_0 \pm \varkappa^2\sqrt{\varkappa^2 - k^2}\frac{k^3}{D_2}\cos^3\varphi_0 +\right.$$
$$\left.+\frac{1}{\tilde{A}_{00}}\left(\frac{\pi\nu}{\varkappa} - \varkappa\sqrt{\varkappa^2 - k^2}\frac{\tilde{B}_0}{D_0}\right)\left(\pi L_1(\varphi_0) - k^2\frac{\tilde{B}_0}{D_0}\cos^2\varphi_0\right) + \ldots\right\}$$
$$L_1(\varphi_0) = k^4\cos^4\varphi_0 - k_0^4$$

The function L is defined in (1.6).

It follows from the second formula of (4.4) that the terms containing $1/\tilde{A}_{00}$ are correction terms with respect to the small parameter ka.

For the case of the scattering of a surface wave

$$U^i = \exp\left(i\varkappa x - \sqrt{\varkappa^2 - k^2 y}\right) \tag{4.6}$$

the asymptotics have the form

$$\Psi(\varphi) = \frac{1}{2\pi}\frac{\varkappa k \sin\varphi}{\nu L(\varphi)}\left\{\varkappa\sqrt{\varkappa^2 - k^2}\,\frac{k^2}{D_0}\cos^2\varphi - \varkappa^2\sqrt{\varkappa^2 - k^2}\,\frac{k^3}{D_2}\cos^3\varphi + \right.$$
$$\left. + \frac{1}{\tilde{A}_{00}}\left(\frac{\pi\nu}{\varkappa} - \varkappa\sqrt{\varkappa^2 - k^2}\,\frac{\tilde{B}_0}{D_0}\right)\left(\pi L_1(\varphi) - k^2\frac{\tilde{B}_0}{D_0}\cos^2\varphi\right) + \ldots\right\}$$
$$A_{\pm} = i\frac{\varkappa(\varkappa^2 - k^2)}{(5\varkappa^4 - 4\varkappa^2 k^2 - k_0^4)\nu}\left\{\frac{\varkappa^2}{D_0} \pm \frac{\varkappa^4}{D_2} + \right.$$
$$\left. + \frac{1}{\tilde{A}_{00}}\left(\frac{\pi\nu}{\varkappa^2\sqrt{\varkappa^2 - k^2}} - \frac{\tilde{B}_0}{D_0}\right)\left(\pi\varkappa^4 - \frac{\tilde{B}_0}{D_0}\varkappa^2 - \pi k_0^4\right) + \ldots\right\} \tag{4.7}$$

The asymptotic expansions (4.5) and (4.7) for the fundamental characteristics of the scattered field contain special integrals I, D_0, D_2 and $\tilde{B}_0$, which depend on the parameters of the plate and of the acoustic medium. In the case of a thin plate ($kh \ll 1$) the integrals can be simplified and clearer formulae are obtained which are given in the next section.

5. DISCUSSION OF THE RESULTS AND CONCLUSION

Note that the leading terms with respect to the parameter ka of the first asymptotic expansion (4.5) are identical with the asymptotic form obtained in [1] for the point-crack model. The correction terms depend on the crack width $2a$ and we will analyse their contribution to the asymptotic form below.

When constructing the asymptotic forms of the scattered field we assumed that only one parameter (ka) is small. The remaining parameters of the system were assumed to be of the order of unity. With these assumptions and for an arbitrary angle of incidence φ_0, the characteristics of the field scattered by a narrow crack turned out to be close to the characteristics of the field scattered by a point crack. At the same time, if a plane wave is incident orthogonally on the plate, the two leading terms in (4.5) disappear and terms of the order of $1/\ln(ka)$ make the main contributions to the asymptotic form. Hence, the point-crack approximation cannot be used in the case of the incidence of a plane wave at angles close to $\pi/2$ (namely, when $|\varphi_0 - \pi/2| < 1\sqrt{(|\ln(ka)|)}$).

As has been shown, the coefficients of the asymptotic expansions depend on the parameters of the plate and of the acoustic medium. We will introduce the dimensionless parameters

$$p = 12(1 - \sigma^2)\rho_0 c^2 / E, \quad d = \rho_0 / \rho$$

where c is the velocity of sound in the acoustic half-space, while the remaining quantities were introduced in Section 1. In terms of these parameters and the parameter $\varepsilon = kh$ we have

$$\nu h^5 = p\varepsilon^2, \quad k_0^4 h^4 = p / d\varepsilon^2$$

The asymptotic expansions of the characteristics of the scattered field for the point model when $\varepsilon \ll 1$ were obtained in [1]. It can be shown that the formulae derived in [1] hold if

$$\varepsilon \ll d^{5/2} p^{-1/2}, \quad \varepsilon \ll p^{1/3} \tag{5.1}$$

The main factor when obtaining these asymptotic forms is the analysis of the dispersion equation

$$l(\lambda)h^5 = \left((\lambda h)^4 - p / d\varepsilon^2\right)\sqrt{(\lambda h)^2 - \varepsilon^2} - p\varepsilon^2 = 0$$

for small ε assuming that the parameters p and d are of the order of unity. In this case the roots of the symbol $l(\lambda)$ on both sheets of the two-sheet Riemann surface λ are situated approximately at the vertices of a right pentagon

$$\varkappa_j = \pm h^{-1} p^{1/5} e^{2\pi i j/5} \varepsilon^{2/5}, \quad j = 0, 1, 2, 3, 4 \tag{5.2}$$

The root $\varkappa_0$ is the wave number of the surface wave and was denoted above by $\varkappa$. Using (5.2) the integrals occurring in the asymptotic form (4.5) can be evaluated by reduction to the sum of residues (see [1])

$$I = -\frac{3\pi i}{5\nu}, \quad \tilde{B}_0 = \nu^{-2/5}\pi\left(\frac{i}{5} + \frac{\sqrt{2}}{50}\left(3\sqrt{5-\sqrt{5}} - \sqrt{5+\sqrt{5}}\right)\right)$$
$$D_0 = \frac{2i}{5\nu^{4/5}}\left(1 - e^{2\pi i/5}\right)^{-1}, \quad D_2 = \frac{2i}{5\nu^{2/5}}\left(1 - e^{6\pi i/5}\right)^{-1} \tag{5.3}$$

Substituting (5.3) into the first asymptotic expansion (4.5), it can be seen that the term proportional to $\tilde{A}_{00}^{-1}$ and which gives the correction with respect to the parameter ka is the leading term with respect to the parameter ε. In fact, the ratio of the patterns in the point model and in the narrow-crack model is of the order of $\varepsilon^{8/5} \ln(ka)$. For the amplitudes of the surface waves, the term containing $\tilde{A}_{00}^{-1}$ is also the leading term with respect to the parameter ε. Hence, even for non-orthogonal incidence the terms corresponding to the point model of the crack only predominate for very small values of ka, namely, $ka \ll \exp(-\varepsilon^{-8/5})$ in the asymptotic form of the diagram and $ka \ll (-\varepsilon^{-4/5})$ in the asymptotic form of the surface-wave amplitudes. The correctness of the model investigated here for such narrow cracks is doubtful and requires additional investigation. However, if $ka > \varepsilon = kh$, the narrow-crack model can be used but the point-crack model obviously cannot.

For non-exponentially small ka terms which are not present in the point model make the main contribution to the asymptotic forms of the scattered-field characteristics. These terms are as follows:

$$\Psi(\varphi, \varphi_0) = 5\frac{\varepsilon^2}{d^2}\frac{\pi - 5iA}{B^2}\sin\varphi \sin\varphi_0, \quad A_{\pm} = 2\pi\frac{\varepsilon}{d}e^{\pi i/5}\frac{\pi i + 5A}{B^2}\sin\varphi_0, \quad B^2 = \pi^2 + 25A^2 \tag{5.4}$$

It can be verified that the asymptotic forms (5.4) satisfy the optical theorem [12]. Calculations show that the major part of the energy scattered by the crack is carried by the surface waves. The effective scattering cross-section has the asymptotic form

$$\Sigma(\varphi_0) = \frac{20\pi^2}{B^2}\frac{\varepsilon h}{d^2}\sin^2\varphi_0 \tag{5.5}$$

The analysis of the applicability of the point model of a crack given above was based on a consideration of the excitation of a spatial-wave system. We will now consider the case of the incidence of a surface wave. The asymptotic form for $A_{\pm}$ in (4.7) leads to the expression

$$A_{\pm} = \frac{1}{2}\left(1 - e^{2\pi i/5}\right) \pm \frac{1}{2}\left(1 - e^{6\pi i/5}\right) - \pi e^{9\pi i/10}\frac{\pi i + 5A}{B^2} \tag{5.6}$$

The first two terms correspond to the point model and are leading terms. The correction in (5.6) is $1/|\ln(ka)|$ times less. Hence, the point-crack model gives correct asymptotic forms of the surface wave process.

The asymptotic forms were investigated assuming that (5.1) holds. If this is not the case, a numerical calculation of the integrals D_0, D_2, $\tilde{B}_0$ and I will be necessary.

REFERENCES

1. KOUZOV, D. P., Diffraction of a plane hydroacoustic wave by a crack in an elastic plate. *Prikl. Mat. Mekh.*, 1963, **27**, 6, 1037–1043.
2. KOUZOV, D. P., The phenomenon of resonance when a plane hydroacoustic wave is diffracted by a system of cracks in an elastic plate. *Prikl. Mat. Mekh.*, 1964, **28**, 3, 409–417.
3. KONOVALYUK, I. P. and KRASIL'NIKOV, V. N., The effect of a stiffener on the reflection of a plane sound wave from a thin plate. In *Problems of Wave Diffraction and Propagation*. Izd. Leningrad. Gos. Univ., Leningrad, 1965, Vol. 4, pp. 149–165.
4. CRIGHTON, D. G. and INNES, D., Low frequency acoustic radiation and vibration response of locally excited fluid-loaded structures. *J. Sound Vibr.*, 1983, **92**, 2, 293–314.
5. SHENDEROV, Ye. L., *Wave Problems of Hydroacoustics*. Sudostroyeniye, Leningrad, 1972.
6. BELINSKII, B. P., Diffraction of a plane wave by a plate reinforced by a protruding stiffener. *Prikl. Mat. Mekh.*, 1978, **42**, 3, 486–493.
7. LANDAU, L. D. and LIFSHITS, Ye. M., *Theoretical Physics*, Vol. 7. *Theory of Elasticity*. Nauka, Moscow, 1965.
8. VOROVICH, I. M., ALEKSANDROV, V. M. and BABESHKO, V. A., *Non-classical Mixed Problems of the Theory of Elasticity*. Nauka, Moscow, 1974.
9. ABRAMOWITZ, M. and STEGUN, I., *Handbook of Special Functions*. Dover, New York, 1975.
10. ANDRONOV, I. V., The scattering of a flexural wave by a finite rectilinear crack in an elastic plate. *Prikl. Mat. Mekh.*, 1990, **54**, 2, 312–321.
11. KANTOROVICH, L. V. and KRYLOV, V. I., *Approximate Methods of Higher Analysis*. Gostekhizdat, Moscow, 1950.
12. BELINSKII, B. P. and KOUZOV, D. P., The optical theorem for a plate-liquid system. *Akust. Zh.*, 1980, **26**, 13–19.

Translated by R.C.G.